Buyer - Vendor incentive inventory model with fixed lifetime product with fixed and linear back orders

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Abstract

The cooperation strategy followed by the buyer and vendor results is an overall effect on the saving percentage along with the impact of inventory costs such as holding cost, fixed and linear backorder cost. This paper deals with buyer - vendor incentive inventory model with fixed lifetime product with fixed and linear back order cost. A distinguishing feature of this model is that both fixed and linear backorder costs are included, whereas previous work include without backorders. The model is developed and proved with a numerical example. The outcome of increasing holding cost, backorder cost for both buyer and vendor or individually is found.

Key words: Inventory, Quantity discount, Coordination, Fixed life time products, Fixed backorder cost, Linear back order cost

1. INTRODUCTION

The vendor - buyer situation is very unique in the business field and the production inventory decision is also very crucial in such situations. The buyer and vendor at times may face the situation of unfulfilled demand which is known as back order. The cost incurred by a business when it is unable to fill an order and must complete it later. A backorder cost can be discrete, as in the cost to replace a specific piece of inventory, or intangible, such as the effects of poor customer service. Backorder costs are usually computed and displayed on a per-unit basis. Backorder costs are important for companies to track, as the relationship between holding costs of inventory and backorder costs will determine whether a company should over- or under-produced. If the carrying cost of inventory is less than backorder costs, the company should over-produce and keep an inventory. The real and perceived costs of the inability to fulfill an order is the backorder cost. The costs can include negative customer relations, interest expenses, etc. The cost incurred by the vendor and buyer due to backorder is termed as backorder cost. The backorder cost is further classified as fixed and linear back order cost.

There will be a huge inventory loss in the case of health care industry, chemical industry, food and beverage industry due to perishability of either raw materials or finished product. The inventory cost that includes ordering cost, carrying cost, and shortage cost also increases due to difficulties in managing the perishable products and therefore there will be a customer dissatisfaction due to the cost and quality deterioration which spoils the image of the company. Liu and Shi (1999) classified perishability and deteriorating inventory models into two major categories namely decay models and finite life time models. The first model deals with the inventory that deteriorates and reduces in quantity continuously in proportion with time. The second model assumes a limited life time for each item. It is further classified into two subcategories namely fixed finite lifetime model and random finite
lifetime model. Fixed life time items model deals with the perishable items while random life time model deals with probability distribution such as exponential and Erlang distribution. Fries (1975), Nandakumar and Morton (1993), Liu and Lian (1999), Lian and Liu (2001), developed the inventory models for fixed life time perishable problem. These researchers have mainly addressed single stage inventory system. Fujiwara et al (1997) studies the problem of ordering and issuing policies in controlling finite life time products, Kanchana and Anulark (2006) analyzed the effect of product perishability and retailers stock out policy of the inventory system. L.H. Chen and F.S. Kang (2010) analyzed the coordination inventory models for the vendor and buyer for trade credit items.

Supply chain management provides an important role for active cooperation and closed coordination therefore few mechanisms are applied to coordinate between parties. Some examples of these mechanisms are quantity discount, revenue sharing, sales rebate and trade credit. The quantity discount is a commonly used scheme among these mechanisms. Goyal and Gupta (1989) reviewed the literatures on the quantity discount model. Yongrui and Jianwen (2010) had researches on buyer-vendor inventory coordination with quantity discount for fixed life time products. We extend the model to consider discount and two types of back order costs to compare with Yongrui and Jianwen (2010). Cardenas-Barron (2011) and G.P. Sphicas (2006) developed an inventory model with fixed and linear backorder. W.K. Wong et al. (2009) analyzed supply chains with sales rebate contracts inventory model. Giannoccaro and Pontrandolfo (2004) developed supply chain coordination by revenue sharing contracts model.

Past researchers analyzed a single-vendor, single-buyer supply chain with fixed life time product without shortages. In this paper vendor, buyer supply chain with two types of back order costs is considered. The developed models analyze the benefit of coordinating supply chain by quantity discount strategy. If the coordination is not considered, given buyer’s economic order quantity, the vendor’s order size is an integer multiple of the buyer’s that minimizes his own inventory cost. The vendor request the buyer to modify his current EOQ under the proposed coordination strategy and the vendor’s order size is another integer multiple of the buyer’s new order quantity. Now the vendor can benefit from lower setup, ordering and inventory holding cost. If the buyer accepts this offer, the vendor must compensate the buyer for his increased inventory cost and possibly provide an additional saving by offering the buyer a quantity discount, which depends on his order size. If we ignore backorders then we get the model by Yongrui and Jianwen (2010), which is considered a particular case in our model.

This paper deals with the holding cost, fixed and linear back order cost for both buyer and vendor. The impact of increase in holding cost of the vendor when fixed and linear cost remains constant is determined using the model. The saving percentage is affected due to the changes in the vendor’s holding cost. The holding cost may increase for both the buyer and the vendor at the same time when fixed and linear remains same, which has an effect over the saving percentage. The effect is determined through the model.

Another situation dealt in this paper is the holding cost increase for the buyer alone where the fixed and linear back order cost is the same. The major aspect is the impact of increase in fixed and linear back order cost when the holding cost for buyer and vendor remains the same. Also the outcome on saving percentage in case increase in fixed back order cost where linear back order cost alone is also analyzed. The situation when holding cost increases either for buyer or vendor when back order also increase is considered for analysis. The result of saving percentage when holding cost for both buyer and vendor and fixed, linear back order cost increases is also determined.

The detailed description of this paper is as follows. In section 2, assumptions and notations, decentralized models with and without coordination models are given. Analytically easily understandable solutions are obtained in these models. It is proved that the quantity discount is the best strategy to achieve
system optimization and win–win outcome. In section 3, a numerical example, algorithm and flow chart are given in detail to illustrate the models. Finally conclusion and summary are presented.

2. MODEL FORMULATION

In this section we analyzed decentralized models with and without coordination. In the coordination strategy quantity discount is offered by the vendor to the buyer.

2.1 ASSUMPTIONS AND NOTATIONS

2.1.1 Assumptions

(1) Demand rate is constant and known over the horizon planning

(2) Back orders are allowed and all back orders are satisfied.

(3) Two types of back order costs are considered. Linear back order cost (back order cost is applied to average backorders) and fixed cost (back order cost is applied to maximum back order level allowed).

(4) Lead time is zero.

(5) The model is for single product.

(6) All items ordered by the vendor arrive fresh and new. i.e., their age equals zero.

2.1.2 Notations

D : Annual demand of the buyer
L : Life time of product
\(k_1, k_2\) : Vendor and buyer’s setup costs per order, respectively
\(h_1, h_2\) : Vendor and buyer’s holding costs, respectively
\(p_1, p_2\) : Delivered unit price paid by the vendor and the buyer respectively
B : Size of back orders in units.
\(\Pi\) : Back order cost per unit (fixed back order cost)
\(\Pi_1\) : Back order cost per unit, per unit of time (linear back order cost)
m : Vendor’s order multiple in the absence of any coordination
n : Vendor’s order multiple under coordination
K : Buyer’s order multiple under coordination. \(KQ_0\) buyer’s new order quantity
d(K) : Denotes the per unit dollar discount to the buyer if he orders \(KQ_0\) every time

2.2 Model 1: EOQ model without coordination with fixed and linear backorders

Without coordination the buyer’s total cost is formulated as follows

\[
TC_b (Q, B) = \frac{Dk_2}{Q} + \frac{(Q-B)^2h_2}{2Q} + \frac{\pi_1B^2}{2Q} + \frac{\pi DB}{Q}
\]

For optimality \(\frac{\partial TC_b (Q,B)}{\partial Q} = 0\) and \(\frac{\partial TC_b (Q,B)}{\partial B} = 0\)

Now \(Q_0^* = \sqrt{\frac{2Dk_2(h_2+\pi_1)-\pi_1^2D^2}{h_2\pi_1}}\) and

\(B_0^* = \frac{h_2Q^*-\pi D}{h_2+\pi_1}\)

Total minimum cost of buyer

\[
TC_b^0 = \frac{1}{h_2+\pi_1} \left(\sqrt{2Dk_2h_2\pi_1(h_2+\pi_1) - \pi_1^2D^2} + \pi D h_2\right)
\]

Without any coordination, the buyer’s order quantity is \(Q_0 = \sqrt{\frac{2Dk_2(h_2+\pi_1)-\pi_1^2D^2}{h_2\pi_1}}\) with the annual cost

\[
TC_b = \frac{1}{h_2+\pi_1} \left(\sqrt{2Dk_2h_2\pi_1(h_2+\pi_1) - \pi_1^2D^2} + \pi D h_2\right).
\]
The vendor’s order size is $mQ_0$, since he faced with a stream of demands at fixed intervals 

$$t_0 = \sqrt{\frac{2k_2 (h_2 + \pi_1) - \pi^2 D}{Dh_2 \pi_1}}.$$ 

The average inventory for vendor’s is 

$$[(m-1)Q_0 + (m-2)Q_0 + \ldots Q_0 + 0Q_0] / m = (m-1)Q_0 / 2.$$ 

In the absence of coordination total annual cost for the vendor is 

1. Procurement cost = $\frac{Dk_1}{mQ_0}$ plus 
2. The annual average inventory holding cost = $\frac{\pi_1 (m-1)B^2}{2Q_0}$ 
3. Linear back order cost = $\frac{\pi_1 (m-1)B^2}{2Q_0}$ plus 
4. Fixed back order cost = $\frac{\pi DB}{Q_0}$ 

Thus 

$$T_{CV}(m) = \frac{Dk_1}{mQ_0} + \frac{(m-1)(Q-B)^2h_1}{2Q_0} + \frac{\pi_1 (m-1)B^2}{2Q_0} + \frac{\pi DB}{Q_0}.$$ 

Without coordination vendor’s problem can be developed as 

$$\min T_{CV}(m) \quad \text{w.r.t} \quad \begin{cases} mt_0 \leq L, \\ m \geq 1, \end{cases}$$ 

where $mt_0 \leq L$ which shows that items are not overdue before they are sold up by the buyer. 

**Theorem 1** 

If $L^2 \geq \frac{2k_2 (h_2 + \pi_1) - \pi^2 D}{Dh_2 \pi_1}$, then 

$$m^* = \min \left\{ \left( \sqrt{\frac{2Dk_1}{(h_2 + \pi_1)^2} + \frac{1}{4}}, \frac{L}{\sqrt{2k_2 (h_2 + \pi_1) - \pi^2 D}} \right) \right\}$$ 

where $m^*$ be the optimum of (2) and $[x]$ is the least integer greater than or equal to $x$, 

$$L^2 \geq \frac{2k_2 (h_2 + \pi_1) - \pi^2 D}{Dh_2 \pi_1}$$ is to ensure that $m^* \geq 1$. 

**Proof** 

Since $T_{CV}(m)$ is strictly convex in $m$ we have 

$$\frac{d^2 T_{CV}(m)}{dm^2} = \frac{2Dk_1}{m^3} \sqrt{2Dk_2 (h_2 + \pi_1) - \pi^2 D^2} > 0.$$ 

Assume that $m_1^*$ is an optimum of (2), then we have 

$$m_1^* = \max \{ \min \{m / T_{CV}(m) \leq T_{CV}(m + 1) \}, 1 \}$$ 

$$= \max \{ \min \{m / m (m + 1) \geq \frac{2Dk_1}{(Q-B)^2h_1 + \pi_1 B^2} \}, 1 \}$$ 

$$= \min \left\{ \left( \sqrt{\frac{2Dk_1}{(h_2 + \pi_1)^2} + \frac{1}{4}}, \frac{L}{\sqrt{2k_2 (h_2 + \pi_1) - \pi^2 D}} \right) \right\}.$$ 

Applying $t_0 = \sqrt{\frac{2k_2 (h_2 + \pi_1) - \pi^2 D}{Dh_2 \pi_1}}$ into the constraints in (1) of equation (2), then there exits the following inequality holds. 

$$m \geq \frac{2k_2 (h_2 + \pi_1) - \pi^2 D}{Dh_2 \pi_1} \leq L.$$ 

Consider $m_2^* = \frac{L}{\sqrt{2k_2 (h_2 + \pi_1) - \pi^2 D}}$, then $m^* = m_1^*$, otherwise 

$m^* = m_2^*$ where $T_{CV}(m)$ is a convex function. 

Therefore if $L^2 \geq \frac{2k_2 (h_2 + \pi_1) - \pi^2 D}{Dh_2 \pi_1}$, then 

$$m^* = \min \left\{ \left( \sqrt{\frac{2Dk_1}{(h_2 + \pi_1)^2} + \frac{1}{4}}, \frac{L}{\sqrt{2k_2 (h_2 + \pi_1) - \pi^2 D}} \right) \right\}.$$ 

**Remrk 1:** Without any coordination the vendor places 

$$m^* = \left( \sqrt{\frac{2Dk_2 (h_2 + \pi_1) - \pi^2 D^2}{h_2 \pi_1}} \right)$$ 

with a regular interval $\frac{2Dk_2 (h_2 + \pi_1) - \pi^2 D^2}{h_2 \pi_1}$. 


The vendor order size is
\[ m^* \left( \frac{2DK_2(h_2+\pi_1)-\pi^2D^2}{h_2\pi_1} \right) \]
and the minimized total cost is \( TC_v(m^*) \).

**Model 2: EOQ model with coordination with fixed and linear backorders**

On coordination strategy i.e., under quantity discount the vendor allows the buyer to change his current order size by \( KQ_0 \) with discount factor \( d(K) \), \( K \) is the positive integer. The vendor’s new order quantity is \( nKQ_0 \), where \( n > 0 \).

The total annual cost for the vendor is

1. Procurement cost = \( \frac{DK_1}{nKQ_0} \) plus
2. The annual average inventory holding cost = \( \frac{(n-1)K(Q-B)^2h_1}{2Q_0} \) plus
3. Linear back order cost = \( \frac{\pi_1(n-1)KB^2}{2Q_0} \) plus
4. Fixed back order cost = \( \frac{pDB}{Q_0} \) plus
5. The buyer’s quantity discount which is equal to \( Dd(K)p_2 \)

Therefore \( TC_v(n) = \frac{DK_1}{nKQ_0} + \frac{(n-1)K(Q-B)^2h_1}{2Q_0} + \frac{\pi_1(n-1)KB^2}{2Q_0} + \frac{pDB}{Q_0} + Dd(K)p_2 \) \( (4) \)

Under coordination the problem can be developed as

\[ \min_{n} TC_v(n) \] \( w.r.t \]

\[ \frac{nKt_0 \leq L,}{Dk_2 + \frac{K(Q-B)^2h_2}{2Q_0} + \frac{\pi_1KB^2}{2Q_0} + \frac{pDB}{Q_0} - \frac{1}{h_2+\pi_1} \left( \sqrt{2DK_2h_2\pi_1(h_2+\pi_1)} - \pi^2D^2 + pDh_2 \right) \leq p_2Dd(K)}{n \geq 1} \]

where \( nKt_0 \leq L \) specifies that items are not overdue before they are sold up by the buyer. The next constraint specifies that the buyer’s cost under coordination is less than the absence of any coordination.

**Theorem 2**

Let \( m^* \) be the optimum of (2) and \( n^* \) be the optimum of (5) then

\[ TC_v(n^*) \leq TC_v(m^*) \] \( (6) \)

**Proof**

The quantity discount factor \( p_2Dd(K) \) is a compensation given by the vendor to the buyer which is a part of the vendor’s costs. If constraint of (5) is an equation then \( TC_v(n) \) is minimum.

\[ d(K) = \frac{Dk_2 + \frac{K(Q-B)^2h_2}{2Q_0} + \frac{\pi_1KB^2}{2Q_0} + \frac{pDB}{Q_0} - \frac{1}{h_2+\pi_1} \left( \sqrt{2DK_2h_2\pi_1(h_2+\pi_1)} - \pi^2D^2 + pDh_2 \right) - p_2D}{p_2D} \] \( (7) \)

Put \( K = 1 \) in (7), we have \( d(1) = 0 \)

i.e.,

\[ \frac{1}{h_2+\pi_1} \left( \sqrt{2DK_2h_2\pi_1(h_2+\pi_1)} - \pi^2D^2 + pDh_2 \right) \]

If we have \( K = 1 \) in (5) we arrive (2) which is the special case of (5). Hence the inequality is true.

**Remark 2**: The above theorem proves that the buyer’s cost with coordination is less than the without coordination. Therefore the vendor gets more benefited for the buyer’s new order quantity.

Now we have find vendor and buyer optimal ordering quantity

Applying \( d(K) \) into (4) we get

\[ TC_v(n) = \frac{DK_1}{nKQ_0} + \frac{(n-1)K(Q-B)^2h_1}{2Q_0} + \frac{\pi_1(n-1)KB^2}{2Q_0} + \frac{pDB}{Q_0} + Dd(K)p_2 \]

\[ + p_2D \left( \frac{Dk_2 + \frac{K(Q-B)^2h_2}{2Q_0} + \frac{\pi_1KB^2}{2Q_0} + \frac{pDB}{Q_0} - \frac{1}{h_2+\pi_1} \left( \sqrt{2DK_2h_2\pi_1(h_2+\pi_1)} - \pi^2D^2 + pDh_2 \right) - p_2D}{p_2D} \right) \] \( (8) \)

Since (8) is a convex function, \( d(K) \) is convex in \( K \).
Consider the minimum of $T \bar{C}_v(n)$ is $K^*$. For optimality $\frac{df}{dn} C_v(n) = 0$, we get

$$K^*(n) = \frac{2D \left( \frac{h_1}{n} + k_2 \right)}{\sqrt{(Q_0 - B)^2[(n-1)h_1 + h_2 + \pi_1 n B^2]}}$$

(9)

Now put

$$K^*(n) = \frac{2D \left( \frac{h_1}{n} + k_2 \right)}{\sqrt{(Q_0 - B)^2[(n-1)h_1 + h_2 + \pi_1 n B^2]}}$$

and

$$t_0 = \frac{2k_2 (h_2 + \pi_1) - \pi^2 D}{\partial h_2 \pi_1}$$

into the first constraint of (5) we get

$$2n^2 \left( \frac{h_1}{n} + k_2 \right) \left[ 2k_2 (h_2 + \pi_1) - \pi^2 D \right] \leq L^2 h_2 \pi_1 \left( \frac{(s_0 + \pi_2) - \pi^2 D}{h_2} \right) \left[ (n-1)h_1 + h_2 + \pi_1 n B^2 \right]$$

Consider

$$f(n) = [-4k_2^2h_2(h_2 + \pi_1) - 2k_2 h_2 \pi^2 D]n^2 + \left[ 2h_2 \pi_1 (\pi_1 B + n B^2) + L^2 h_2 \pi_1 \pi^2 D^2 - 4k_2 h_2 (h_2 + \pi_1) + 2k_2 h_2 \pi^2 D \right]n + L^2 \pi_1 (h_2 - h_1) (\pi_1 B + n B^2)$$

(10)

then $n k t_0 \leq L$ is equivalent $f(n) \geq 0$.

Apply $K^*(n)$ and $t_0$ as

$$\frac{\sqrt{2k_2 (h_2 + \pi_1) - \pi^2 D}}{\partial h_2 \pi_1}$$

into $T \bar{C}_v(n)$, we get

$$T \bar{C}_v(n) = \frac{\sqrt{2k_2 (h_2 + \pi_1) - \pi^2 D}}{\partial h_2 \pi_1}$$

(11)

Therefore (5) is equivalent to

$$\min T \bar{C}_v(n)$$

w.r.t

$$f(n) \geq 0$$

$$n \geq 1$$

(12)

Since $\sqrt{x}$ is a strictly increasing function for $x \geq 0$, (12) is equivalent to

$$\min T \bar{C}_v(n) = \frac{h_2 n D}{2k_2 D (h_2 + \pi_1)^2 - \pi^2 D} \left[ \left( \frac{k_1}{n} + k_2 \right) \left( (Q_0 - B)^2[(n-1)h_1 + h_2 + \pi_1 n B^2] + 2 \pi^2 D^2 B^2 \right) \right]$$

w.r.t

$$f(n) \geq 0$$

$$n \geq 1$$

(13)

We must discuss the properties of $T \bar{C}_v(n)$ and $f(n)$, to solve the above equation.

Since $T \bar{C}_v(n)$ is convex when $h_2 \geq h_1$, because

$$T \bar{C}_v(n) = \frac{h_2 n D}{2k_2 D (h_2 + \pi_1)^2 - \pi^2 D} \left[ \left( \frac{k_1}{n} + k_2 \right) \left( (Q_0 - B)^2[(n-1)h_1 + h_2 + \pi_1 n B^2] + 2 \pi^2 D^2 B^2 \right) \right] > 0$$

otherwise it is concave.

$f(n)$ is strictly concave because

$$f'(n) = -2[4k_2^2h_2(h_2 + \pi_1) - 2k_2 h_2 \pi^2 D] < 0.$$

**Proposition 1**

Assume that the minimum of $\bar{C}_v(n)$ is $n_1^*$ for $n \geq 1$, then we have

$$n_1^* = \begin{cases} \frac{k_1 (h_2 - h_1) (s_0 + n B^2)}{\sqrt{k_2 (s_0 + h_2 + h_1) (s_0 + n B^2)}} & 0 < 1 - \frac{k_1 (h_2 - h_1) (s_0 + n B^2)^2}{k_2 (s_0 + h_2 + h_1) (s_0 + n B^2)^2} \leq 1 \\ \frac{k_1 (h_2 - h_1) (s_0 + n B^2)}{\sqrt{k_2 (s_0 + h_2 + h_1) (s_0 + n B^2)}} & \text{otherwise} \end{cases}$$

(14)

**Proof**

$T \bar{C}_v(n_1^*) \leq \min\{T \bar{C}_v(n_1^* - 1), T \bar{C}_v(n_1^* + 1)\}$ is true if $n_1^*$ is the minimum of $\bar{C}_v(n)$, $n \geq 1$.

Now $T \bar{C}_v(n_1^*) - T \bar{C}_v(n_1^* - 1) = \frac{k_2 h_1 - k_1 (h_2 - h_1)}{n_1^* (n_1^* - 1)} + \pi_1 k_2 B^2 \leq 0$

we have,

$$\left( n_1^* - \frac{1}{2} \right) \leq \frac{k_1 (h_2 - h_1)(s_0 + n B^2)}{k_2 (s_0 + h_2 + h_1) (s_0 + n B^2)} + \frac{1}{4}$$

(15)

Similarly, by $T \bar{C}_v(n_1^*) - T \bar{C}_v(n_1^* + 1) \leq 0$

we have

$$\left( n_1^* + \frac{1}{2} \right) \geq \frac{k_1 (h_2 - h_1)(s_0 + n B^2)}{k_2 (s_0 + h_2 + h_1) (s_0 + n B^2)} + \frac{1}{4}$$

(16)

Hence $n_1^* = \left[ \frac{k_1 (h_2 - h_1)(s_0 + n B^2)}{k_2 (s_0 + h_2 + h_1) (s_0 + n B^2)} + \frac{1}{4} \right]^{1/2}$

when

$$\frac{k_1 (h_2 - h_1)(s_0 + n B^2)}{k_2 (s_0 + h_2 + h_1) (s_0 + n B^2)} + \frac{1}{4} \leq n_1^* \leq \frac{k_1 (h_2 - h_1)(s_0 + n B^2)}{k_2 (s_0 + h_2 + h_1) (s_0 + n B^2)} + \frac{1}{4} + \frac{1}{2}$$

(14), (15), (16).

Otherwise $n_1^* = 1$ when

$$\frac{k_1 (h_2 - h_1)(s_0 + n B^2)}{k_2 (s_0 + h_2 + h_1) (s_0 + n B^2)} + \frac{1}{4} < 0.$$}

Also note that

$$n_1^* = 1, 0 < \frac{k_1 (h_2 - h_1)(s_0 + n B^2)}{k_2 (s_0 + h_2 + h_1) (s_0 + n B^2)} < 2.$$}

Therefore (14) is true.

**Proposition 2**

Let $n_{2(1)}$ and $n_{2(2)}$ be solution of (10), then

i) If $Y^2 + 4XZ < 0$ or $Y^2 + 4XZ \geq 0$ and

$n_{2(1)}^* < 1$, then $f(n) < 0$ for $n \geq 1$. 

ii) If $Y^2 + 4XZ \geq 0$ and $n_{2(1)}^* \geq 1$, then If $n_{2(2)}^* = 1$, f(n) $\geq 0$ for $[n_{2(2)}^*] \leq n \leq n_{2(1)}^*$.
If $n_{2(2)}^* < 1$ and $n_{2(1)}^* \geq 1$, f(n) $\geq 0$ for $1 \leq n \leq n_{2(1)}^*$

where $X = -4k_2^2h_2(h_2 + \pi_1) - 2k_2h_2\pi^2 D$, $Y = L^2\pi_1h_2(\pi_1B + \pi D)^2 + L^2\pi_1^2h_2^2B^2 - 4k_1k_2h_2(h_2 + \pi_1) + 2k_1h_2\pi^2 D$ and $Z = L^2\pi_1(h_2 - h_1)(\pi_1B + \pi D)^2$

Proof

Now solve the quadratic equation $f(n) = 0$, we get

$$n_{2(1)}^* = \frac{\sqrt{Y^2 + 4XZ} + Y}{2X} \quad \text{and} \quad n_{2(2)}^* = \frac{-\sqrt{Y^2 + 4XZ} - Y}{2X}$$

The following conclusions are true because f(n) is an quadratic equation.

1) f(n) $< 0$ where $Y^2 + 4XZ < 0$ for every n.
2) $n_{2(1)}^*$ and $n_{2(2)}^*$ are real solutions of f(n) = 0 where $Y^2 + 4XZ \geq 0$. In view of $n \geq 1$,

i) $f(n) < 0$ where $n_{2(1)}^* < 1$, for $n \geq 1$;

ii) $f(n) \geq 0$ where $n_{2(2)}^* \geq 1$, for $[n_{2(2)}^*] \leq n \leq n_{2(1)}^*$;

iii) $f(n) \geq 0$ where $n_{2(2)}^* < 1$ and $n_{2(1)}^* \geq 1$, for $1 \leq n \leq n_{2(1)}^*$.

Remark 3: If $[n_{2(2)}^*] \leq n \leq [n_{2(1)}^*]$ or $1 \leq n \leq [n_{2(1)}^*]$ then the conclusion (ii) of proposition 2 is true and the first constraint of (5) does not true for any $n \geq 1$.

Theorem 3

i) $n^* = n_{11}^*$ if $1 \leq n_{11}^* \leq [n_{2(1)}^*]$.

ii) $n^* = [n_{2(1)}^*]$ if $n_{11}^* > [n_{2(1)}^*]$ where $h_2 \geq h_1$ and $n_{2(2)}^* \geq 1$.

Proof

Since $n_{11}^*$ is the minimum of $TC_v(n)$ for $n \geq 1$ then $TC_v(n)$ is a convex function.

Hence if $n^* = n_{11}^*$, $1 \leq n_{11}^* \leq [n_{2(1)}^*]$ else $n^* = [n_{2(1)}^*]$, $n_{11}^* > [n_{2(1)}^*]$.

Here $TC_v(n)$ is decreasing on the interval $n_{11}^* > [n_{2(1)}^*]$ so $n^* = [n_{2(1)}^*]$.

Remark 4: $TC_v(n)$ is strictly concave if the vendor’s unit holding cost is higher than the buyer’s. This is not common in practice so we will not give further discussion about this.

Theorem 4

If $h_2 \geq h_1$ then $K^*(n^*) > 1$.

Proof

$$K^*(n) = \sqrt{\frac{2D(\frac{k_1}{h_1} + k_2)}{(Q_0-B)\left[(n-1)h_1 + h_2\right] + n_{11}B^2}}$$

= $$\sqrt{\frac{2D(\frac{k_1}{h_1} + k_2)}{(\frac{h_1}{h_2} + \frac{k_1}{h_1})\left[(n-1)h_1 + h_2\right] + n_{11}B^2}}$$

(1) If \(\frac{k_1(h_2-h_1)(\pi_1B+\pi D)^2}{k_2(\pi_1^2B^2h_2^2 + h_1(\pi_1B+\pi D)^2)} \geq 2\) then $n^* = n_{11}^*$.

i.e., $n^* = n_{11}^* = \left[\frac{-k_1(h_2-h_1)(\pi_1B+\pi D)^2}{2k_2(\pi_1^2B^2h_2^2 + h_1(\pi_1B+\pi D)^2)} + \frac{1}{4} - \frac{1}{2}\right]$.

$K^*(n^*)$ is a decreasing function of n if

$$\left|x + \frac{1}{4} - \frac{1}{2}\right| \leq \sqrt{x} + 1 \text{ is true for } x \geq 0.$$  

To prove $K^* \left[\frac{k_1(h_2-h_1)(\pi_1B+\pi D)^2}{2k_2(\pi_1^2B^2h_2^2 + h_1(\pi_1B+\pi D)^2)} + 1\right] > 1$

i.e.,

$$\frac{2D(\frac{k_1}{h_1} + k_2)}{(Q_0-B)^2\left[(n-1)h_1 + h_2\right] + n_{11}B^2} \geq 2$$

= $$\frac{2D(\frac{k_1}{h_1} + k_2)}{(\frac{h_1}{h_2} + \frac{k_1}{h_1})\left[(n-1)h_1 + h_2\right] + n_{11}B^2} \geq 2$$

> 1
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order, \( k_2 = 100 \) $ per unit, \( \alpha = 0.5 \), \( L = 0.25 \) year, \( k_1 = 300 \) $ per unit.

Examples:
1. Given \( D = 10,000 \) units per year, \( p_2 = 30 \) $ per unit, \( \alpha = 0.5 \), \( L = 0.25 \) year, \( k_1 = 300 \) $ per order, \( k_2 = 100 \) $, \( \pi = 0.01 \), \( \pi_1 = 1.0 \). The different values of \( h_1 \), \( h_2 \) and computational results are as specified in Table 1.

2. Given \( D = 10,000 \) units per year, \( p_2 = 30 \) $ per unit, \( \alpha = 0.5 \), \( L = 0.25 \) year, \( k_1 = 300 \) $ per order, \( k_2 = 100 \) $, \( h_1 = 5 \), \( h_2 = 10 \). The different values of \( \pi \), \( \pi_1 \) and computational results are as specified in Table 2.

3. Given \( D = 10,000 \) units per year, \( p_2 = 30 \) $ per unit, \( \alpha = 0.5 \), \( L = 0.25 \) year, \( k_1 = 300 \) $ per order, \( k_2 = 100 \) $, \( h_1 = 5 \), \( h_2 = 10 \). The different values of \( h_1 \), \( h_2 \), \( \pi_1 \) and computational results are as specified in Table 3.

4. Given \( D = 10,000 \) units per year, \( p_2 = 30 \) $ per unit, \( \alpha = 0.5 \), \( L = 0.25 \) year, \( k_1 = 300 \) $ per order, \( k_2 = 100 \) $, \( h_1 = 5 \), \( h_2 = 10 \). The different values of \( h_1 \), \( h_2 \), \( \pi_1 \) and computational results are as specified in Table 4.

5. Given \( D = 10,000 \) units per year, \( p_2 = 30 \) $ per unit, \( \alpha = 0.5 \), \( L = 0.25 \) year, \( k_1 = 300 \) $ per order, \( k_2 = 100 \) $, \( h_1 = 5 \), \( h_2 = 10 \). The different values of \( h_1 \), \( h_2 \), \( \pi_1 \) and computational results are as specified in Table 5.

Hence (17) holds if \( k_1 \), \( k_2 \), \( h_1 \), \( h_2 \), \( B \), \( D \), \( \pi_1 \), \( \pi \) are all positive and \( h_2 \geq h_1 \).

Remark 5: Theorem (4) specifies that the buyer’s order size is greater to compare with cooperation against the non-cooperation if \( h_2 \geq h_1 \).

3. NUMERICAL EXAMPLES

In this section, numerical examples are presented to illustrate the performance of the quantity discount strategy proposed in previous section. The sensitivity analysis of cost savings on parameters has been given.

The buyer’s saving in percentage
\[
SP_b = 100 \times \frac{(TC_v(m^*) - TC_v(n^*))}{TC_v(m^*)}.
\]

The vendor’s saving in percentage
\[
SP_v = 100(1-\alpha)(TC_v(m^*) - TC_v(n^*)) / TC_v(m^*)
\]

The vendor’s saving in percentage if he does not share the saving with the buyer
\[
SP_v = 100(TC_v(m^*) - TC_v(n^*)) / TC_v(m^*)
\]

Examples:
1. Given \( D = 10,000 \) units per year, \( p_2 = 30 \) $ per unit, \( \alpha = 0.5 \), \( L = 0.25 \) year, \( k_1 = 300 \) $ per order, \( k_2 = 100 \) $, \( \pi = 0.01 \), \( \pi_1 = 1.0 \). The

different values of \( h_1 \), \( h_2 \) and computational results are as specified in Table 1.

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5. Given \( D = 10,000 \) units per year, \( p_2 = 30 \) $ per unit, \( \alpha = 0.5 \), \( L = 0.25 \) year, \( k_1 = 300 \) $ per order, \( k_2 = 100 \) $, \( h_1 = 5 \), \( h_2 = 10 \). The different values of \( h_1 \), \( h_2 \), \( \pi_1 \) and computational results are as specified in Table 5.

Tables: Given in bottom of manuscript

The crux of the model is

1. Savings percentage remains constant when holding cost for vendor increases and back order costs are constant.

2. Savings percentage increases when holding cost of buyer increases when back order costs are constant.

3. Savings percentage increases when holding cost for both buyer and vendor increases and back order costs remains constant.

4. Savings percentage decreases when linear back order cost increases and holding cost and fixed backorder cost remains same.

5. Savings percentage decreases when fixed back order increases and holding cost and linear backorder cost remains same.
6. Savings percentage decreases when back order costs are increased and holding cost remains constant.

7. Savings percentage decreases when backorder cost and holding cost of either vendor or buyer increases.

8. Savings percentage decreases when both holding cost and backorder cost increases.

The computational result indicates that the system can save more cost under coordination.

3.1 ALGORITHM AND FLOWCHART

3.1.1: Algorithm

Step 1: Initialize the values

Step 2: Read the values

Step 3: Find $Q_0$, $B_0$

Step 4: If $\frac{k_1(h_2-h_1)(x_1B+\pi D)^2}{k_2(x_1B^2 h_2^2+h_1(x_1B+\pi D)^2)} \geq 2$ then go to step 5 otherwise go to step 6

Step 5:

i) Find $n^*$ by using the equation first constrain of (14)

ii) Find $K^*(n)$ by using the equation (9)

iii) Find $TC_V(n^*)$ by using the equation (8)

iv) Find optimum $m^*$ by using the equation (3)

v) Find $TC_V(m^*)$ by using (2)

vi) Find $TC_b$

vii) Calculate $SP_b$, $SP_{v_1}$, $SP_{v_2}$

Step 6: i) Initialize $n = 1$ then

ii) Go to step 5(ii) – (vii)

Step 7: End

Figures: Given in bottom of manuscript

4. CONCLUSION

In this paper, we have developed inventory model in which quantity discount coordination strategy with linear and fixed backorder cost for a buyer - vendor supply chain of fixed life time product. Analytically proved and optimized decisions are arrived using the model. The cooperation strategy always increases the saving percentage of both vendor and the buyer. The various situations of changes especially increase in holding cost, backorder cost both linear as well as fixed is dealt in the paper. The model brings into light that the saving percentage increases when holding cost increase for both buyer and vendor or buyer alone and backorder cost remains the same. This situation is beneficial to both buyer and vendor and increase the profit for them. There is no change in saving percentage when vendor alone increases holding cost and backorder cost remains the same. The increase in backorder cost in any form either for the vendor or buyer or both leads to a decrease in saving percentage. Thus it could be concluded with numerical proof that an increase in backorder cost will reduce the benefit for both buyer and vendor with coordination. Another useful finding of the study is that the impact of an increase in backorder cost is much higher than the impact of increasing in holding cost. Thus backorder cost both fixed and linear costs are equally important from the cost and benefit point of view for the buyer and vendor. It has been proved that the buyer’s order size is higher with cooperation than the non cooperation. The vendor gives order size dependent on discount to the buyer to compensate his increased inventory cost. We prove that the decentralized quantity discount strategy can achieve system optimization and win-win outcome. As a result both the vendor and the buyer benefit in the long run. Numerical example is presented to illustrate the model. Even though well consider the backorder cost, the system cost is reduced in comparison with the model by Yongrui and Jianwen (2010). Hence we obtained more savings for both the vendor and the buyer in our models with and without coordination.
REFERENCES


### Table 1: Computational results for different values of $h_1$ and $h_2$

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### Table 2: Computational results for different values of $\pi$ and $\pi_1$

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Table 4: Computational results for different values of $h_2$, $\pi$ and $\pi_1$

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Table 5: Computational results for different values of $h_1$, $h_2$, $\pi$, and $\pi_1$

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<td>$\pi$</td>
<td>$\pi_1$</td>
<td>$d(K)$</td>
<td>$Q$</td>
<td>$B$</td>
<td>$SP_b$</td>
<td>$SP_{v1}$</td>
<td>$SP_{v2}$</td>
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Table 5: Computational results for different values of $h_1$, $h_2$, $\pi$ and $\pi_1$.
FIGURES

Figure 1(a): Effect of changes when holding cost for vendor increase (Table 1)

Figure 1(b): Effect of changes when holding cost for vendor and buyer increases (Table 1)

Figure 1(c): Effect of changes when holding cost for buyer increase (Table 1)

Figure 2(a): Effect of changes when linear backorder cost increase (Table 2)

Figure 2(b): Effect of changes when fixed backorder cost increases (Table 2)

Figure 2(c): Effect of changes when both fixed and linear backorder cost increase (Table 2)