UNSTEADY MAGNETOHYDRODYNAMIC FLOW BETWEEN TWO PARALLEL PLATE THROUGH A POROUS MEDIUM

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Abstract

In this paper we discuss an analytical solution of unsteady MHD flow between two parallel plates through a porous medium with uniform suction in the lower plate and uniform injection in the upper plate. External uniform magnetic field are applied perpendicularly to the plates while the fluid motion is subjected to an exponential axial and transverse velocity and pressure gradient. The solution of the problem is obtained with the help of similarity transformation. The exact solution of the velocity in the porous medium is analytically derived, its behaviour computationally discussed with reference to the various governing parameters. The axial and transverse velocity of the fluid and the pressure distribution are presented. Analytical expression is given for the velocity field of the fluid and the effects of the various parameters entering into the problem are discussed with the help of graphs.

Key words: Fluid flow, Parallel Plates, MHD flow, Porous Medium, Similarity trandformation,Pressure distribution.

Nomenclature

ρ	–	Density of the fluid
h	–	Height of the channel
K	–	Permeability of the porous medium
μ	–	Coefficient of viscosity
ψ	–	Stream function
σ	–	Electrical conductivity of the fluid
B_0	–	Electromagnetic induction
H_0	–	Transverse magnetic field
u	–	Axial component of the velocity
v	–	Transverse Component of the velocity
η	–	Dimensionless distance

I. INTRODUCTION

The MHD flow between two parallel plates is called Hartmann flow. It has many Applications in MHD power generators, MHD pumps, aerodynamic heating. Hartmann and Lazarus [1] studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite plates. Then a lot of research work concerning the Hartmann flow has been obtained under different physical effects. The Study of flow has been carried out by several authors. Many researchers have reported that the flow is electrically conducting fluid [2-6]. MHD is the fluid mechanics of electrically conducting fluids, some of these fluids include liquid metals such as mercury, molten iron etc.,and ironized gases known as physicists as Plasma, one example being the solar atmosphere. If an electrically conducting fluid is placed in a constant magnetic field, the motion of the fluid induces current which create forces on the fluid. The governing equations that have been solved either analytically or numerically.

The requirements of modern technology have stimulated the interest in fluid flow studies, which involve the interaction of several phenomena. One such study is presented, when a viscous fluid flows over a porous surface, because of its importance in many engineering problems such as flow of liquid in a porous bearing (Joseph and Tao [7]), in the field of water in river beds, in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs, in chemical engineering for filtration and purifications process. Cunningham and Williams [8] also reported several geophysical applications of flow in porous medium, viz. porous rollers and its natural occurrence in the flow of rivers through porous banks and beds and the flow of oil through underground porous rocks. The mathematical theory of the flow of fluid through a porous medium was initiated by Darcy [9]. For the steady flow, he assumed that viscous forces were in equilibrium with external forces due to pressure difference and body forces. Later on Brinkman [13] proposed modification of the Darcy's law for porous medium. In most of the examples, the fluid flows through porous medium, have two regions. In region I, the fluid is free to flow and in region II, the fluid flows through the porous medium. To link flows in two regions certain, matching conditions are required at the interface of two regions. This type of couple flow, with different geometry and with several kinds of matching conditions, has been examined by several authors, viz. William [10] and Ochoa-Tapia *et al.* [11]. Srivastava *et al.* [12] discussed the flow and heat transfer of a viscous fluid confined between a rotating plate and a porous medium, by assuming that the flow in the porous medium was governed by Brinkman equation [13] and that in the free flow region by the Navier-Stokes equations. The problem (in which the liquid occupies the semi-infinite region on one side of the disk and the motion is axially symmetric) of steady forced flow of an incompressible viscous fluid against a rotating disk was studied by Schlichting *et al.*[14]. *Ibid*[15], has analysed Momentum transfer at the boundary between a porous medium and a homogeneous fluid – II A complete review of this paper and also same related work has been given by Moore [16]. Recently, Chaudhary *et al.* [17] discussed the flow of viscous incompressible fluid confined between a rotating disk and a porous medium.

The two dimensional steady state laminar flow in channels with porous walls has numerous applications in various branches of Engineering and Technology such as boundary layer control and transpiration cooling problems. It plays an important role in the study of problems which involve diffusion phenomena in a flowing gas stream. Berman[18] was the first researcher who studied the problem of steady flow of an incompressible viscous fluid through a porous channel with rectangular cross section,when the Reynolds number is low. He obtained a perturbation solution assuming normal wall velocities to be equal. Then Sellars[19] extended the problem studied by Berman when the Reynolds number is very high. Afterwards Yuan[20] and Terill[21] studied the problem for various values of suction and injection Reynolds numbers. Terill and Shrestha[22] have analysed the same problem, Laminar flow through parallel and uniformly porous walls of different permeability .Drake[23] has considered the flow of an incompressible viscous fluid in a long channel of rectangular section due to a periodic pressure gradient. Ganesh.S, Krishnambal[24] analysed Magnetohydrodynamic flow of viscous fluid between two parallel porous plates.

II. ASSUMPTIONS

1. The plates are porous.
2. Flow is between non conducting parallel plates.
3. MHD flow is considered.
4. Viscosity and density of the fluid is considered to be constant.
5. u and v are axial and transverse velocity components in the direction of x and y respectively.

III. GENERAL SOLUTION TO THE PROBLEM

The flow of an incompressible viscous fluid between two parallel porous plates $y = 0$ and $y = h$ is considered in the presence of a transverse magnetic field which is applied perpendicular to the walls in a parallel plate channel bounded by a loosely packed porous medium. The fluid is driven by a uniform pressure gradient parallel to the channel plates. Let u and v be the velocity components in the x and y directions respectively at time t in the flow field.

The equation of continuity is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ ...(1)

Navier Stoke Equations are :

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + u\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma_e B_0^2 u - \frac{\mu u}{K} \quad ...(2)$$

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad ...(3)$$

By the discussions in the previous sections, Let us choose the solutions of the equations (1)-(3) respectively as

$$u = u(x, y)\, e^{-nt} \quad ...(4)$$

$$v = v(x, y)\, e^{-nt}$$

$$p = p(x, y)\, e^{-nt} \quad ...(4)$$

With the boundary conditions

$$\left.\begin{array}{ll} u(x,0)=0 & u(x,h)=0 \\ v(x,0)=v_1 & v(x,h)=v_2 \end{array}\right\} \quad ...(5)$$

Let the stream functions are

$$u(x,y)=\frac{\partial \psi}{\partial y} \text{ and } v(x,y)=-\frac{\partial \psi}{\partial x} \quad ...(6)$$

From equations (2), (3) and (6), we have

$$-n\rho\frac{\partial \psi}{\partial y}=-\frac{\partial p}{\partial x}+\mu\frac{\partial}{\partial y}(\nabla^2\psi)$$

$$-\sigma_e B_0^2\frac{\partial \psi}{\partial y}-\frac{\mu}{K}\frac{\partial \pi}{\partial y} \quad ...(7)$$

$$n\rho\frac{\partial \psi}{\partial x}=-\frac{\partial p}{\partial y}-\mu\frac{\partial}{\partial x}(\nabla^2\psi) \quad ...(8)$$

Differentiating equations (7) & (8) with respect to 'y' & 'x' partially, we get

$$\frac{\partial^2 p}{\partial x\partial y}=\mu\frac{\partial^2}{\partial y^2}(\nabla^2\psi)+n\rho\frac{\partial^2\psi}{\partial y^2}$$

$$-\sigma_e B_0^2\frac{\partial^2\psi}{\partial y^2}-\frac{\mu\,\partial^2\psi}{K\partial y^2} \quad ...(9)$$

$$\frac{\partial^2 p}{\partial x\, dex\, y}=-\mu\frac{\partial^2}{\partial x^2}(\nabla^2\psi)-n\rho\frac{\partial^2\psi}{\partial x} \quad ...(10)$$

From (9) and (10), we have

$$\left[\nabla^2-\left(\frac{\sigma_e B_0^2 u+(\mu/K)-\rho\, n}{\mu}\right)\right]\nabla^2\psi=0 \quad ...(11)$$

The equation of continuity can be satisfied by a stream function of the form

$$\psi(x,\eta)=h\left(\frac{u_0}{a}-\frac{v_2 x}{h}\right)f(\eta) \quad ...(12)$$

where u_0 is the average entrance velocity and $\eta=\frac{y}{h}$ is a dimensionless distance.

Here $a=1-\frac{v_1}{v_2}, 0\le v_1\le v_2$

Substituting (12) in (11), we have

$$f^{iv}(\eta)-\alpha^2 h^2 f''(\eta)=0 \quad ...(13)$$

where

$$a^2=\frac{\sigma_e B_0^2 u+(\mu/K)-\rho\, n}{\mu}$$

IV. MATHEMATICAL SOLUTION TO THE PROBLEM

Equation (13) reduces to the form

$$(D^4-\alpha^2 h^2 D^2)f(\eta)='0 \quad ...(14)$$

with the boundary conditions

$$\left.\begin{array}{ll} f(0)=1-a & f(1)=1 \\ f'(0)=0, & f'(1)=0 \end{array}\right\} \quad ...(15)$$

Hence the solution of (14) subjecting to the boundary condition (15) is

$$f(\eta)=\frac{2}{2\alpha h\sinh\alpha h+4(1-\cosh\alpha h)}\times$$

$$\left[\begin{array}{l} 4(1-\cosh(\alpha h))-2a(1-\cosh(\alpha h)) \\ -2(1-a)\alpha h\sinh(\alpha h)+2a\alpha h\eta\sinh(\alpha h) \\ +2a\cosh(\alpha h(\eta-1)-2a\cosh(\alpha h\eta) \end{array}\right] \quad ...(16)$$

Substituting the value of in the stream function

$$\psi(x,\eta)=h\left(\frac{u_0}{a}-\frac{v_2 x}{h}\right)f(\eta) \quad ...(17)$$

Hence the Axial Velocity of the Fluid

$$u=u(x,y)\,e^{-nt}$$

$$=\frac{\partial\psi}{\partial y}e^{-nt}$$

$$=\left(\frac{u_0}{a}-\frac{v_2 x}{h}\right)\times e^{-nt}\times$$

$$\left[\frac{2\alpha ha\sinh(\alpha h)+2a\alpha h\sinh(\alpha(y-h)-2a\alpha h\sinh\alpha y)}{2\alpha h\sinh\alpha h+4(1-\cosh(\alpha h))}\right] \quad ...(18)$$

The Transverse Velocity of the Fluid

$$v=v(x,y)\,e^{-nt}$$

$$=-\frac{\partial\psi}{\partial x}$$

$$= \frac{v_2 e^{-nt}}{2\alpha h \sinh \alpha h + 4(1 - \cosh \alpha h)}$$

$$\left[\begin{array}{c} 4(1 - \cos h(\alpha h)) - 2a(1 - \cos h(\alpha h)) - \\ 2(1-a)\alpha h \sinh(\alpha h) + 2a\alpha y \sin h(\alpha h) + \\ 2a \cos h(\alpha(y-h)) - 2a \cos h(\alpha y) \end{array} \right] \quad \dots (19)$$

V. PRESSURE DISTRIBUTIOIN

The Pressure Drop can be obtained from (7), (8) and (12)

$$p(x,\eta) - p(0,0) = \left(\frac{u_0 x}{a} - \frac{v_2 x^2}{2h} \right) \times$$

$$\left(\frac{\mu}{h^2} f'''(\eta) - \left(\frac{\mu}{K} + \sigma_e B_0^2 u - n\rho f'(\eta) \right) \right)$$

$$+ v_2 h \left(\left(n\rho \sum_0^{\eta} f(\eta)\, d\eta \right) + \left(\frac{\mu}{h^2} \int_0^{\eta} f'(\eta)\, d\eta \right) \right) \dots (20)$$

VI. GRAPHICAL REPRESENTATION

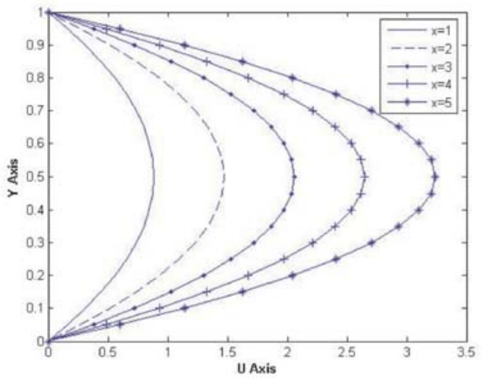

Fig. 1. Axial Velocity of the Fluid for various values of x

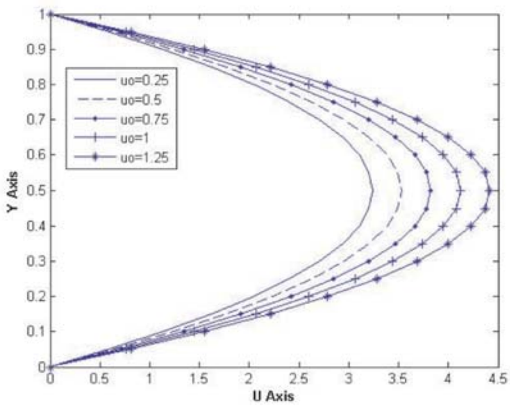

Fig. 2. Axial Velocity of the Fluid for different values of u0

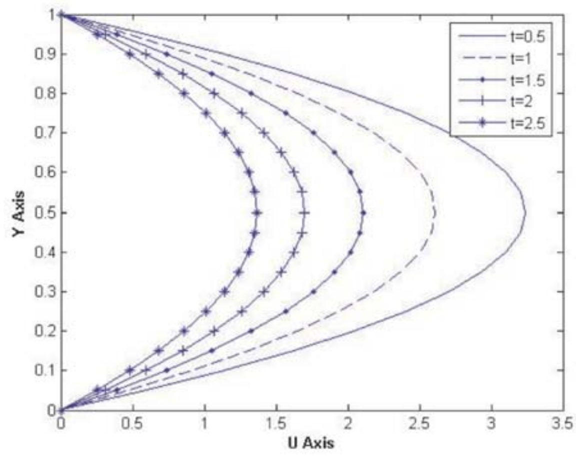

Fig. 3. Axial Velocity of the Fluid for different values of t

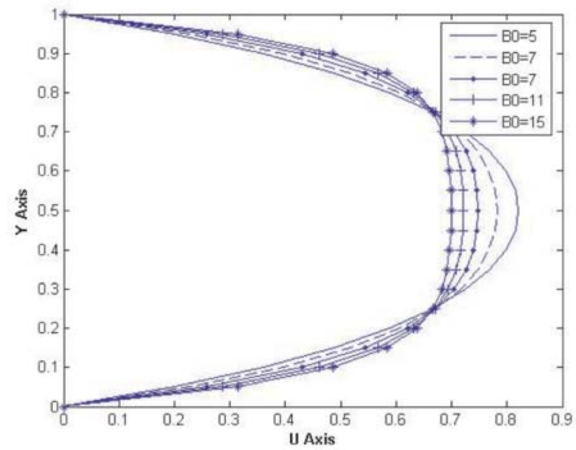

Fig. 4. Axial Velocity of the Fluid for different values of B_0

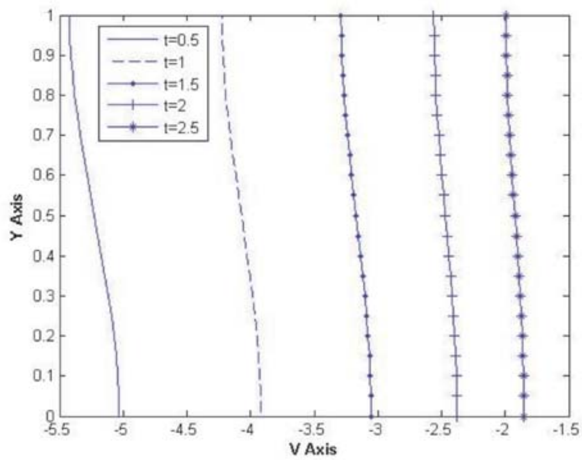

Fig. 5. Transverse Velocity of the Fluid for different values of t

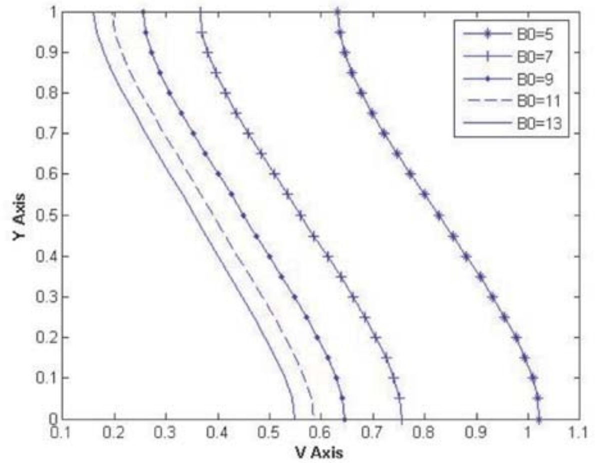

Fig. 6. Transverse Velocity of the Fluid for different values of Bo

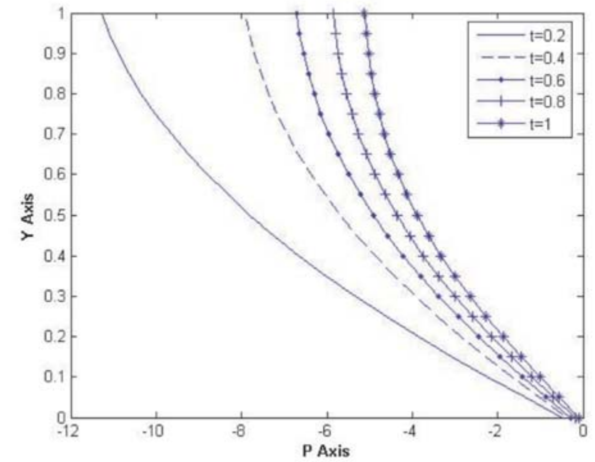

Fig. 7. Pressure Distribution for different values of t

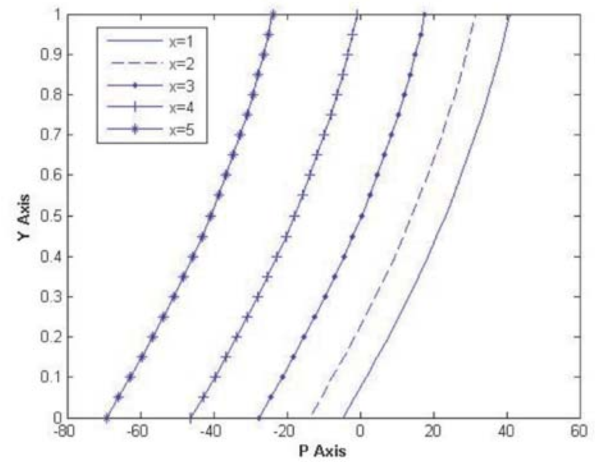

Fig. 8. Pressure Distribution when x increases

VII. RESULT AND DISCUSSIONS

This paper analyses the performance of fluid subject to various parameters. Figure 1&2 shows that the velocity of the fluid increases when x,u_0 increases. Figure 3 shows that velocity of the fluid decreases as t increases. Figure 4 shows that velocity of the fluid decreases when y lies between 0.2 & 0.8 as B_0 increases. Figure 5 & 6 shows that the transverse velocity of the fluid increases when t,B_0 increases. Figure 7 shows that pressure of the fluid increases as t increases. Figure 8 shows that pressure of the fluid decreases as x increases. Figure 5,6 & 8 shows that the magnitude of the upper plate and lower plate are same. Figure 7 shows that the magnitude of the upper is higher than the magnitude of the lower plate..

The above results reduce to the results of [24] when K is infinity.

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