Optimum Production Coordination Inventory Model without Shortages

K. Sivakumar[1] P. Muniappan[2]

[1]Professor, Sathyabama University, Chennai, Tamil Nadu, Chennai-119, India.
[2]Research Scholar, Sathyabama University, Chennai, Tamil Nadu, Chennai-119, India.
email: [1]siva111k@gmail.com

Abstract

This paper mainly deals with optimum production coordination inventory model without shortages for perishable products. The decentralized model with and without coordination to analyze the benefit costs of both the retailer and supplier for coordinating the supply chain is proposed in this paper. The quantity discount is implemented for coordinating the supply chain. The paper also includes a detailed numerical example for more understanding of the proposed strategy.

Key words: Inventory, Quantity discount, Coordination, Perishable products.

I. INTRODUCTION

The inventory cost that includes ordering cost, carrying cost and production cost also increases due to difficulties in managing the perishable products. Rather than the cost, quality deterioration spoils the image of the company and acts as a cause for customer dissatisfaction. Perishable products have limited life time for each items. Fires (1975), Nandakumar and Mortan (1993), Liu and Lian (1999), Lian and Liu (2001), developed the inventory models for fixed life time perishable problem. Fujiwers et al (1997) studies the problem of ordering and issuing policies in controlling finite life time products, Kanchana and Anulark (2006) analysed the effect of product preishability and retailers stock out policy of the inventory system.

In supply chain management, quantity discount is one of the mechanisms to coordinate between supplier and retailer. Goyal and Gupta (1989) reviewed the literatures on the quantity discount model. Yongrui and Jianwen (2010) had researches on buyer-vendor inventory coordination with quantity discount for fixed life time products. We extend the model to consider production to compare with Yongrui and Jianwen (2010). Chen and Kang (2010) developed Coordination between vendor and buyer considering trade credit and items of imperfect quality. Giannoccaro and Pontrandolfo (2004) studied supply chain coordination by revenue sharing contracts. Liu and Shi (1999) considered (s,S) model for inventory with exponential life times and renewal demands. Wong, Qi and Leung (2009) concentrate coordinating supply chains with sales rebate contracts and vendor-managed inventory. Past researchers analyzed a single-vendor, single-buyer supply chain with fixed life time product without shortages. In this paper supplier and retailer supply chain without shortage and supplier's production is considered. The developed models analyze the benefit of coordinating supply chain by quantity discount strategy.

The detailed description of the paper is as follows. Assumption and notations are given in section 2. In section 3, Decentralized models with and without coordination models are formulated. Analytically easily understandable solutions are obtained in these models. It is proved that the quantity discount is the best strategy to achieve system optimization and win – win outcome. In section 4, a numerical example is given in detail to illustrate the models. Finally conclusion and summary are presented.

II. ASSUMPTIONS AND NOTATIONS

2.1 Assumptions

1. Demand is known and constant.
2. Shortages are not allowed.
3. Lead time is zero.
4. During the production run the production of the item is continuous and at a constant rate until production of quantity Q is complete.

2.2 Notations

P Production rate per year ($P > D$)

D Annual demand of the retailer

L Life time of product

k_1, k_2 Supplier and retailer's setup costs per order, respectively

h_1, h_2 Supplier and retailer's holding costs, respectively

p_1, p_2 Delivered unit price paid by the supplier and the retailer respectively

Q_0 Retailer's EOQ

m Supplier's order multiple in the absence of any coordination

n Supplier's order multiple under coordination

K Retailer's order multiple under coordination.

d(K) Denotes the per unit dollar discount to the retailer if he orders K(Q_0) every time.

III. MODEL FORMULATION

In this section, decentralized models with and without coordination are analyzed. Quantity discount is offered by the supplier in the model with coordination.

Case 1 Model formulation without coordination

In the absence of any coordination, the retailer's order quantity is $Q_0 = \sqrt{\frac{2D k_2}{b_2}}$ with the annual cost $TC^r = \sqrt{2Dk_2 h_2}$. The supplier's order size should be some integer multiple of Q_0 denote by mQ_0, since he faced with a stream of demands at fixed intervals $t_0 = Q_0/D$. In this case, the supplier's average inventory held up per year is [(m–1) Q_1 + (m–2) Q_1+…….. Q_1+0 Q_1] / m = (m–1) Q_1/2

Now the total annual cost for the supplier is given by

$$TC_s\ (m) = \frac{Dk_1}{mQ_0} + \frac{(m-1)\, h_1\, Q_0}{2}\left(1-\frac{D}{P}\right)$$

$$= \frac{k_1}{m}\sqrt{\frac{Dh_2}{2k_2}} + (m-1)\left(1-\frac{D}{P}\right)\sqrt{\frac{Dk_2}{2h_2}}\ h_1$$

So the supplier's problem without coordination can be formulated as follows

Min TC_s (m)

$$\text{s.t} \begin{cases} mt_0 \le L, \\ m \ge 1, \end{cases} \qquad (1)$$

where $mt_0 \le L$ is to ensure that items are not overdue before they are used up (sold up) by the retailer.

Theorem 1

Let m* be the optimum of (1) if $L^2 \ge \frac{2k^2}{Dh^2}$, then

$m^* =$

$$\min\left\{\left\lceil \sqrt{\frac{k_1 h_2}{\left(1-\frac{D}{P}\right)k_2 h_1}+\frac{1}{4}} - \frac{1}{2}\right\rceil, \left\lceil \frac{L}{\sqrt{\frac{2k_2}{Dk_2}}}\right\rceil\right\}, \qquad (2)$$

where $\lceil x \rceil$ is the least integer greater than or equal to x, $L^2 \ge \frac{2k_2}{Dh_2}$ is to ensure that $m^* \ge 1$

Proof

TC_s (m) is strictly convex in m. Since $\frac{d^2\,TC_v(m)}{dm^2} = \frac{k_1}{m^3}\sqrt{\frac{2Dh_2}{k_2}} > 0$. Let m_1* be the optimum of min Tc_s (m) then

$$m_1^* = \max\{\min\{m/TCs(m) \le TC_s(m) \le TC_s(m+1)\}, 1\}$$

$$= \max\left\{\min\left\{m/m(m+1) \ge \frac{2Dk_1}{Q_0^2\left(1-\frac{D}{P}\right)h_1}\right\}, 1\right\}$$

$$= \left\lceil \frac{h_2 k_1}{h_1 k_2\left(1-\frac{D}{P}\right)} + \frac{1}{4} - \frac{1}{2}\right\rceil \ge 1.$$

Substituting $t_0 = \sqrt{\frac{2k_2}{Dh_2}}$ into the constraints in (1), the following inequality holds.

$m\sqrt{\frac{2k_2}{Dh_2}} \le L.$ Set $m_2^* = \frac{L}{\sqrt{\frac{2k^2}{Dh_2}}}.$ Since $L^2 \ge \frac{2k_2}{Dh_2}, m_2^* \ge 1$ holds.

In view of TC_S (m) is a convex function, if $m_1^* \le m_2^*, m^* = m_1^*,$ else $m^* = m_2^*.$ So if $L^2 \ge \frac{2k_2}{Dh_2}, m^* =$ min $\{ m_1^*, m_2^* \}$. The proof of theorem 1 is complete.

Remark 1: In the absence of any coordination the supplier's order size is $m^*\sqrt{\frac{2Dk_2}{h_2}}$ and place $\frac{D}{m^*\sqrt{\frac{2Dk_2}{h_2}}}$ orders each year with an interval $m^*\frac{\sqrt{\frac{2Dk_2}{h_2}}}{D}$ throughout that time. The minimized total cost is $TC_S(m^*)$.

Case 2 Model formulation with coordination

Under the quantity discount coordination strategy, the supplier request the retailer to alter his current order size by a factor K (K > 0) and compensate the buyer a quantity discount at a discount factor d(K). Now the supplier's order quantity is nKQ_0, where n > 0 and KQ_0 is the buyer's new order quantity. The total cost TC_S (n) of the vendor is composed of three parts:

1. The ordering cost which is equal to $\frac{Dk_1}{nKQ_0}$
2. The inventory holding cost which is equal to $\frac{(n-1)\left(1-\frac{D}{P}\right)h_1\, kQ_0}{2}$
3. The retailer's quantity discount which is equal to $Dd(K)\, P_2$

Therefore, (3)

$$TC_S(n) = \frac{Dk_1}{nKQ_0} + \frac{(n-1)\left(1-\frac{D}{P}\right)h_1\, KQ_0}{2} + P_2\, Dd(K)$$

The problem with coordination can be formulated as follows

min $TC_V(n)$

$$\text{subject to } \begin{cases} nKt_0 \le L, \\ \frac{Dk_2}{KQ_0} + \frac{KQ_0h_2}{2} - \sqrt{2Dk_2\, h_2} \le p_2\, Dd(K), \\ n \ge 1, \end{cases} \quad (4)$$

The first constraint $nKt_0 \le L$ is ensuring that items are overdue before they are used up (sold up) by the retailer. The second constraint is the retailer's participation constraint, i.e., the retailer's cost under coordination cannot exceed that in the absence of any coordination.

Theorem 2

Let m^* and n^* be the optimum of (1) and (4) respectively, then the following inequality holds:

$$TC_S(n^*) \le TC_S(m^*) \quad (5)$$

Proof

The term $p_2\, Dd(k)$ in the right hand side of the second constraint of (4) is just the compensation to the retailer by the supplier, which is a component of the supplier's costs. By the second constraint $p_2\, Dd(K)$ takes the smallest value only when the second constraint is an equation, so if $TC_S(n)$ is minimized, the second constraint must be an equation.

i.e., $\frac{Dk_2}{KQ_0} + \frac{KQ_0\, h_2}{2} - \sqrt{2Dk_2\, h_2} = p_2\, Dd(K)$

$$\text{Hence, } d(K) = \frac{\frac{Dk_2}{KQ_0} + \frac{KQ_0\, h_2}{2} - \sqrt{2Dk_2\, h_2}}{p_2\, D} \quad (6)$$

If $K = 1$, then $d(1) = \frac{\sqrt{2Dk_2\, h_2} - \sqrt{2Dk_2\, h_2}}{p_2\, D} = 0$

Hence equation (4) is equivalent to (1) if K = 1, i.e., (1) is a special case of (4), so (5) holds. The proof is complete.

Remark 2

By theorem (2), the optimum total cost under coordination is not greater than that without coordination the supplier will gain by inducing the retailer to order KQ_0 every time.

Theorem 3

Let K^* be the optimum order quantity for retailer,

$$K^* > 0 \quad \text{then} \quad K^*(n) = \frac{1}{Q_0}\sqrt{\frac{2D\left(\frac{k_1}{n} + k_2\right)}{(n-1)\left(1 - \frac{D}{P}\right)h_1 + h_2}}$$

and min

$$\tilde{TC}_s(n) = Dk_1\left(1 - \frac{D}{P}\right)h_1 + \frac{Dk_1\left[h_1 - \left(1 - \frac{D}{P}\right)h_1\right]}{n} +$$

$$nDk_2\left(1 - \frac{D}{P}\right)h_1 + Dk_2\left[h_2 - \left(1 - \frac{D}{P}\right)h_1\right]$$

Proof

Substitute (6) into (3),

$$TC_s(n) =$$

$$\frac{Dk_1}{nkQ_0} + \frac{(n-1)\left(1 - \frac{D}{P}\right)h_1\, kQ_0}{2} +$$

$$P_2 D\left(\frac{\frac{Dk_2}{KQ_0} + \frac{KQ_0\, h_2}{2} - \sqrt{2Dk_2\, h_2}}{P_2 D}\right)$$

Since d (K) is convex in K, $TC_s(n)$ is obviously convex in K.

Let K^* be the minimum of TC_s (n), by a simple calculation,

$$K^*(n) = \frac{1}{Q_0}\sqrt{\frac{2D\left(\frac{k_1}{n} + k_2\right)}{(n-1)\left(1 - \frac{D}{P}\right)h_1 + h_2}} \qquad (8)$$

Now $nKt_0 \le L$, we have

$$\left(\frac{k_1}{n} + k_2\right)n^2 \le \frac{L^2 Q_0^2\, h_2}{4k_2}\left((n-1)\left(1 - \frac{D}{P}\right)h_1 + h_2\right)$$

Set $g(n) = -k_2 n^2 + \left(\left(\frac{DL^2}{2}\right)\left(1 - \frac{D}{P}\right)h_1 - k_1\right)n +$

$$\frac{DL^2}{2}\left(h_2 - \left(1 - \frac{D}{P}\right)h_1\right) \qquad (9)$$

then first constraint of (4) is equivalent to $g(n) \ge 0$.

Substituting (8) and $t_0 = \sqrt{\frac{2k_2}{Dh_2}}$ into TC_s (n),

$$TC_s(n) =$$

$$\frac{\sqrt{2\left[Dk_1\left(1 - \frac{D}{P}\right)h_1 + \frac{Dk_1\left[h_2 - \left(1 - \frac{D}{P}\right)h_1\right]}{n} + nDk_2\left(1 - \frac{D}{P}\right)h_1 + Dk_2\right]}}{\sqrt{2Dh_2\, k_2}} \qquad (10)$$

So (4) is equivalent to

min TC_s (n)

$$\text{subject to} \begin{cases} g(n) \ge 0, \\ n \ge 1, \end{cases} \qquad (11)$$

Since $\sqrt{x}$ is a strictly increasing function for $x \ge 0$, (11) is equivalent to the following problem:

min

$$\tilde{TC}_s(n) = Dk_1\left(1 - \frac{D}{P}\right)h_1 + \frac{Dk_1\left[h_2 - \left(1 - \frac{D}{P}\right)h_1\right]}{n} +$$

$$nDk_2\left(1 - \frac{D}{P}\right)h_1 + Dk_2\left[h_2 - \left(1 - \frac{D}{P}\right)h_1\right]$$

$$\text{subject to} \begin{cases} g(n) \ge 0, \\ n \ge 1, \end{cases} \qquad (12)$$

It is obvious that (12) is a nonlinear programming. To solve (12), we must discuss the properties of $\tilde{TC}_s(n)$ and g (n).

since

$\tilde{TC}_s''(n) = \frac{2Dk_1\left[h_2 - \left(1 - \frac{D}{P}\right)h_1\right]}{n^3} > 0$, $\tilde{TC}_s(n)$ is convex when $h_2 \geq \left(1-\frac{D}{P}\right)h_1$ and concave otherwise. By $g''(n) = -2k_2 < 0$, $g(n)$ is strictly concave. The proof is complete

Theorem 4

Let n_1^* be the minimum of $\tilde{TC}_s(n)$ for $n \geq 1$, then

$$n_1^* = \left\{ \left[\sqrt{\frac{k_1\left[h_2-\left(1-\frac{D}{P}\right)h_1\right]}{k_2\left(1-\frac{D}{P}\right)h_1}+\frac{1}{4}}-\frac{1}{2} \right] \right.$$

1, otherwise

$$\frac{k_1\left[h_2-\left(1-\frac{D}{P}\right)h_1\right]}{k_2\left(1-\frac{D}{P}\right)h_1} \geq 2 \quad (13)$$

Proof

Since n_1^* is the minimum of $\tilde{TC}_s(n)$ for $n \geq 1$, the following inequality holds:

$$\tilde{TC}_s(n_1^*) \leq \min\left\{\tilde{TC}_s(n_1^*-1), \tilde{TC}_s(n_1^*+1)\right\}$$

By $\tilde{TC}_s(n_1^*) - \tilde{TC}_s(n_1^*-1) = \frac{-Dk_1\left[h_2-\left(1-\frac{D}{P}\right)h_1\right]}{n_1^*(n_1^*-1)} +$

$$DK_2\left(1-\frac{D}{P}\right)h_1 \leq 0$$

$$\left(n_1^*-\frac{1}{2}\right)^2 \leq \frac{k_1\left[h_2-\left(1-\frac{D}{P}\right)h_1\right]}{k_2\left(1-\frac{D}{P}\right)h_1}+\frac{1}{4} \quad (14)$$

Similarly, by $\tilde{TC}_s(n_1^*) - \tilde{TC}_s(n_1^*+1) \leq 0$ we have

$$\left(n_1^*+\frac{1}{2}\right)^2 \geq \frac{k_1\left[h_2-\left(1-\frac{D}{P}\right)h_1\right]}{k_2\left(1-\frac{D}{P}\right)h_1}+\frac{1}{4} \quad (15)$$

Hence, if

$$\frac{k_1\left[h_2-\left(1-\frac{D}{P}\right)h_1\right]}{k_2\left(1-\frac{D}{P}\right)h_1}+\frac{1}{4} < 0,\ \tilde{TC}_s(n_i) \leq \tilde{TC}_s(n_i+1)$$

for any given n, so $n_1^* = 1$.

If $\frac{k_1\left[h_2-\left(1-\frac{D}{P}\right)h_1\right]}{K_2\left(1-\frac{D}{P}\right)h_1}+\frac{1}{4} \geq 0$,

by (14) & (15)

$$\sqrt{\frac{k_1\left[h_2-\left(1-\frac{D}{P}\right)h_1\right]}{k_2\left(1-\frac{D}{P}\right)h_1}+\frac{1}{4}}-\frac{1}{2} \leq n_i \leq$$

$$\sqrt{\frac{k_1\left[h_2-\left(1-\frac{D}{P}\right)h_1\right]}{k_2\left(1-\frac{D}{P}\right)h_1}+\frac{1}{4}}+\frac{1}{2}$$

Therefore, $n_1^* = \left[\sqrt{\frac{k_1\left[h_2-\left(1-\frac{D}{P}\right)h_1\right]}{k_2\left(1-\frac{D}{P}\right)h_1}+\frac{1}{4}}-\frac{1}{2}\right]$

Furthermore, note that if

$$0 < \frac{k_1\left[h_2-\left(1-\frac{D}{P}\right)h_1\right]}{k_2\left(1-\frac{D}{P}\right)h_1} < 2,$$

$n_1^* = 1$, so (13) holds. The proof is complete.

NUMERICAL EXAMPLE

In this section, a numerical example is presented to illustrate the performance of the quantity discount strategy proposed in previous sections.

Example

Given D = 10,000 units per year, P = 20000, p_2 = 30\$ per unit, α = 0.5, L = 0.25 year, k_1 = 300\$ per order. The different values of h_1 and computational results are as specified in Table 1.

Table 1. Computational Results

k_2	h_1	h_2	n^*	m^*	K^*	Q^*	$TC_s(m)$	$TC_s(n)$	$TC_r(m)$
100	2	5	3	3	1.1952	632.4555	2213.6	2129.2	3162.3
100	3	5	2	3	1.3868	632.4555	2529.8	253806	3162.3
100	4	5	2	2	1.3363	632.4555	3004.2	2753.8	3162.3
100	5	5	1	2	2.0000	632.4555	3162.3	3162.3	3162.3
100	5	6	2	2	1.3284	577.3503	3319.8	3055.1	3464.1
100	5	7	2	2	1.3572	534.5225	3474.4	3150.4	3741.7
100	6	10	2	3	1.3868	447.2136	3577.9	3590.1	4472.1
100	6	11	2	3	1.4015	426.4014	3624.4	3676.2	4690.4
100	6	15	3	3	1.1952	365.1484	3834.1	3687.9	5777.2
200	2	5	2	2	1.2076	894.4272	2124.3	2008.6	4472.1
200	3	5	1	2	1.5811	894.4272	2347.9	2598.9	4472.1
200	4	5	1	2	1.5811	894.4272	2571.5	2598.9	4472.1
200	5	5	1	1	1.5811	894.4272	3354.1	2598.9	4472.1
200	5	6	1	1	1.5811	816.4966	3674.2	2847.0	4899.0
200	5	7	1	2	1.5811	755.9289	2929.2	3075.1	5291.5
200	6	10	1	2	1.5811	632.4555	3320.4	3675.4	6324.6
200	6	11	2	2	1.1726	603.02227	3392.0	3266.2	6633.2
200	6	15	2	2	1.2076	516.3978	3679.3	3479.0	7746.0

CONCLUSIONS

In this paper we have developed optimum production coordination inventory model without shortages for perishable products in supplier and retailer. Through a numerical example we get analytically easily understandable solutions. It has been proved that the buyer's order size is higher with coordination than the non coordination. The developed model deals with the supplier and retailer get more profit by purchasing a large number of items with the some quantity discount. Especially the supplier get more profit than the retailer when he manufacturing the products. i.e., the developed prove that the decentralized quantity discount strategy can achieve system optimization and win-win outcome. Numerical example is presented to illustrate the model.

REFERENCES

[1] Chen, L.H., Kang, F.S., 2010. Coordination between vendor and buyer considering trade credit and items of imperfect quality. International Journal of Production Economics 123(1), 52–61.

[2] Fries, B., 1975. Optimal order policies for a perishable commodity with fixed lifetime. Operations Research 23(1), 46–61.

[3] Fujiwara, O., Soewandi, H., Sedarage, D., 1997. An optimal and issuing policy for a two-stage inventory system for perishable products. European Journal of Operational Research 99(2), 412–424.

[4] Giannoccaro, I., Pontrandolfo, P.,2004. Supply chain coordination by revenue sharing contracts. International Journal of Production Economics 89(2), 131–139.

[5] Goyal, S.K., Gupta, Y.P., 1989. Integrated inventory models: the buyer–vendor coordination. European Journal of Operational Research 41(3), 261–269.

[6] Kanchana, K., Anulark, T., 2006. An approximate periodic model for fixed-life perishable products in a two-echelon inventory - distribution system. International Journal of Production Economics 100(1), 101–115.

[7] Lian, Z., Liu, L., 2001. Continuous review perishable inventory systems: models and heuristics. IIE Transactions 33(9), 809–822.

[8] Liu, L., Lian , Z.,1999. (s,S) model for inventory with fixed lifetime. Operations Research 47(1), 130–158.

[9] Liu, L., Shi, D., 1999. (s,S) model for inventory with exponential life times and renewal demands. Naval Research Logistics 46(1), 39–56.

[10] Nandakumar, P., Morton, T.E., 1993. Near myopic heuristic for the fixed life perishability problem. Management Science 39(12), 1490–1498.

[11] Wong, W.K., Qi,J., Leung, S.Y.S., 2009. Coordinating supply chains with sales rebate contracts and vendor-managed inventory. International Journal of Production Economics 120(1), 151–161.

[12] Younguri Duan, Jianwen Luo, Jiazhen Huo , 2010. Buyer-vendor inventory coordination with quantity discount incentive for fixed lifetime product. International Journal Production Economics.

Dr. K. Sivakumar, working as a Professor of Mathematics at Sathyabama University for the last 12 years. He has published many papers in International and National Journals and conferences. His area of interests includes Applied Mathematics, Outlier Mining and Statistics.