Edge product Cordial Labeling and Total Magic Cordial Labeling of Regular Digraphs

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Abstract—

In this paper we prove that regular digraphs are Edge product Cordial and Total magic cordial. A digraph G is said to have edge product cordial labeling if there exists a mapping, f:E(G)⟶{0,1} and induced vertex labeling function f* :V(G)⟶{0,1} such that for any vertex vi∈ V(G), f* (vi) is the product of the labels of outgoing edges provided the condition ⎥v(0)- v(1)⎥≤ 1and ⎥e(0)- e(1)⎥≤ 1 is hold where v(i) is the number of vertices of G having label i under f* and e(i) is the number of edges of G having label i under f for i = 0,1. A digraph G is said to have a total magic cordial labeling with constant C if there exists a mapping f : V(G)∪ E(G)⟶{0,1} such that for any vertex vi, the sum of the labels of outgoing edges of vi, and the label of itself is a constant C (mod2) provided the condition ⎥g(0)-g(1)⎥≤1 is hold where g(0) = v(0) + e(0) and g(1) = v(1) + e(1), where v(i),e(i): i∈{0,1}are the number of vertices and edges labeled with i respectively.

Key words: Regular digraph, Graph labeling, Edge product cordial and Total magic cordial labeling

I. INTRODUCTION

The graph labeling was introduced by Rosa in 1965.A graph labeling is an assignment of integers to edges or vertices or both [1]. The concept of cordial labeling was introduced by Cahit in 1987 [2] and in the same paper he investigated several results on this newly introduced concept. A new labeling called E-Cordial was introduced by Yilmaz and Cahit.I [9]. In 2004 Sundaram et al[3] have introduced product cordial labeling in which the absolute difference in cordial labeling is replaced by product of the vertex labels. S.K.Vaidya and C.M.Barasara[4] introduced a variant of product cordial labeling and name it as edge product cordial labeling. Unlike in product cordial labeling the roles of vertices and edges are interchanged.

They proved some results for undirected graph and now we prove it for directed graphs. The original concept of total edge magic graph is due to Kotzig and Rosa [5,6]. R.Rajeswari and R.Parameswari defined total magic cordial labeling for Paley digraphs [10].

In future the interconnection networks of multiprocessor computing systems may be very complex than the today's trend. Interconnection networks are often modeled by digraphs. Regular digraphs are often used to model the interconnection networks, since most of them are vertex transitive.

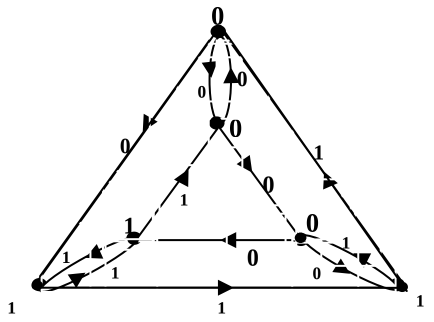
The present paper also aimed to prove the edge product cordial labeling and total magic cordial labeling of regular digraphs which would be useful in the fields of parallel computation and cryptography.

II. PRELIMINARIES

Let us see the basic definitions which are necessary to our investigation.

Definition 2.1

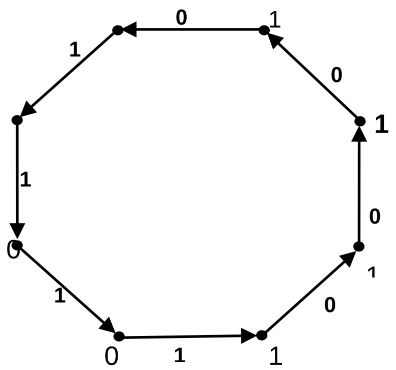
A digraph G is said to have edge product cordial labeling if there exists a mapping, f:E(G)⟶{0,1} and induced vertex labeling function f* :V(G)⟶{0,1} such that for any vertex vi∈ V(G), f* (vi) is the product of the labels of outgoing edges provided the condition ⎥v(0)- v(1)⎥≤ 1and ⎥e(0)- e(1)⎥≤ 1 is hold where v(i) is the number of vertices of G having label i under f* and e(i) is the number of edges of G having label i under f for i = 0,1. See fig 2(a).



2(a).Edge product cordial labeling of S3

Definition 2.2

A digraph G is said to have a total magic cordial labeling with constant C if there exists a mapping f : V(G) ∪ E(G)→{0,1} such that for any vertex vi, the sum of the labels of outgoing edges of vi, and the label of itself is a constant C (mod2) provided the condition │g(0)- g(1)│≤1 is hold where g(0) = v(0) + e(0) and g(1) = v(1) + e(1) where v(i),e(i): i∈{0,1}are the number of vertices and edges labeled with i respectively. See fig 2(b).



2(b). Total magic cordial labeling of C8

Theorem 2.3

The Payley digraph is total magic cordial. Refer [10]

Definition 2.4

A regular directed graph must also satisfy the stronger condition that the in degree and out degree of each vertex are equal to each other.

III. MAIN RESULTS

A. Edge product cordial labeling of regular digraph

In this section we show the existence of edge product cordial labeling for regular digraphs.

Theorem 3.1.2

The m regular digraph with n vertices is Edge product cordial.

Proof:

We know that the m regular digraph G(V,E) has n vertices and every vertex has m outgoing and m incoming arcs. Therefore the digraph totally has mn arcs. Denote the vertex set of G as V = {v1,v2,…,vn} and edge set of G as E ={e11,e12,…e1m,e21e22…e2m… en1…enm} where eij is jth outgoing arc from ith vertex.

To prove G(V,E) admits edge product cordial labeling we have to show that there exists a function f:E(G)⟶{0,1}and induced vertex labeled function f*:V(G)⟶{0,1} such that for every vertex vi , 1≤ i ≤ n

f* (vi) = f(ei1)f(ei2)…f(eim)

and the condition │v(0)- v(1)│≤ 1and │e(0)- e(1)│≤ 1 is hold.

Where v(i) is the number of vertices of G having label i under f* and e(i) is the number of edges of G having label i under f for i = 0,1

If n is even

Define f : E(G)⟶{0,1}as

For 1≤ i ≤ n and j = 1,2…m

$$f(eij) = \begin{cases} 0 \qquad \text{for } 1 \leq \mathrm{i} \leq \frac{\mathrm{n}}{2} \\ 1 \text{ for } \left(\frac{\mathrm{n}}{2}\right) + 1 \ \leq \mathrm{i} \leq \mathrm{n} \end{cases}$$

Then the induced function f* (vi) = f(ei1) f(ei2)…f(eim) for every vertex vi , 1≤ i ≤ n.

$$\text{i.e., } f^* (vi) = \begin{cases} 0 \qquad \text{for } 1 \leq \mathrm{i} \leq \mathrm{n}/2 \\ 1 \text{ for } \left(\frac{\mathrm{n}}{2}\right) + 1 \ \leq \mathrm{i} \leq \mathrm{n} \end{cases}$$

Since n is even the number of edges labeled zero is mn/2 and the number of edges labeled 1 is mn/2 .i.e., e(0) = (mn)/2 and e(1) = (mn)/2 ;│e(0)- e(1)│ = 0 ≤ 1.

Thus the number of vertices labeled zero is n/2 and the number of vertices labeled one is n/2. i.e.,v(0) = n/2 = v(1); │v(0)- v(1)│= 0 ≤ 1.

Therefore the regular m digraph G(V,E) is Edge product cordial, when n is even

If n is odd,

Subcase (i) If m is even

Define f:E(G)→{0,1}as for all j = 1,2,…,m and for $i = 1,2, \dots, n-1$

f(eij) = $\begin{cases} 0 \text{ for } 1 \le i \le (n-1)/2 \\ 1 \text{ for } \frac{n-1}{2} + 1 \le i \le n-1 \end{cases}$

For i = n,

f(enj) = $\begin{cases} 0 & \text{for } j = 1,3,5 \dots m \\ 1 & \text{for } j = 2,4,6 \dots m-1 \end{cases}$

Then the induced function f* (vi) = f(ei1) f(ei2)…f(eim) for every vertex vi, 1≤ i ≤ n.

f*(vi) = $\begin{cases} 0 & \text{for } 1 \le i \le \frac{n-1}{2} \\ 1 & \text{for } \frac{n+1}{2} \le i \le n-1 \\ 0 & \text{for } i = n \end{cases}$

Since m is even, the number of edges labeled zero is mn/2 and the number of edges labeled 1 is mn/2. i.e., e(0) = (mn)/2 and e(1) = (mn)/2 ;│e(0)- e(1)│ = 0 ≤ 1.

Thus the number of vertices labeled zero is (n-1)/2 + 1 and the number of vertices labeled one is (n-1)/2. i.e., v(0) = (n-1)/2 + 1 and v(1) = (n-1)/2;

│v(0)- v(1)│= │(n-1)/2-(n-1)/+1│= 1 ≤ 1.

Subcase (ii) If m is odd

Define f:E(G)→{0,1}as For all j = 1,2,…,m and for $i = 1,2, \dots, n-1$

f(eij) = $\begin{cases} 0 \text{ for } 1 \le i \le (n-1)/2 \\ 1 \text{ for } \frac{n-1}{2} + 1 \le i \le n-1 \end{cases}$

For i = n,

f(enj) = $\begin{cases} 0 \text{ for } j = 1,3,5 \dots m-1 \\ 1 \text{ for } j = 2,4,6 \dots m \end{cases}$

Then the induced function f* (vi) = f(ei1) f(ei2)…f(eim) for every vertex vi, 1≤ i ≤ n.

f*(vi) = $\begin{cases} 0 & \text{for } 1 \le i \le \frac{n-1}{2} \\ 1 & \text{for } \frac{n+1}{2} \le i \le n-1 \\ 0 & \text{for } i = n \end{cases}$

Since m is odd, the number of edges labeled zero is (mn+1)/2 and the number of edges labeled 1 is (mn-1)/2. i.e., e(0) = (mn+1)/2 and e(1) = (mn-1)/2 ;│e(0)- e(1)│ = 1 ≤ 1.Thus the number of vertices labeled zero is (n-1)/2 + 1 and the number of vertices labeled one is (n-1)/2. i.e., v(0) = (n-1)/2 + 1 and v(1) = (n-1)/2 .│v(0)- v(1)│= │(n-1)/2 - (n-1)/+1│= 1 ≤ 1.

Therefore when n is odd, The digraph G(V,E) is edge product cordial for all even and odd number of generators.

Hence the m regular digraph is edge product cordial.

Corollary 3.1.2

The Cayley digraph is edge product cordial.

B. Total magic cordial labeling of regular digraph

In this section we show the existence of total magic cordial labeling for odd regular digraph

Theorem 3.2.1

The m regular digraph is total magic cordial, if $m \equiv 1 \pmod 2$.

Proof:

We know that the m regular digraph G(V,E) has n vertices and every vertex has m outgoing and m incoming arcs, where m is odd. Therefore the digraph totally has mn arcs. Denote the vertex set of G as V = {v1,v2,…,vn} and edge set of G as E ={e11,e12,…e1m,e21e22…e2m… en1…enm} where eij is jth outgoing arc from ith vertex.

To prove the odd regular digraph is total magic cordial, we have to show that for any vertex vi, the sum of the labels of outgoing edges from vi and label of itself is a constant mod2 provided the number of vertices and edges labeled with 0 and the number of vertices and edges labeled with 1 differs at most by 1.

Case (i): When n is even

Define f: V(G)→{0,1}as

f(vi) = $\begin{cases} 0 & \text{for } 1 \le i \le n/2 \\ 1 & \text{for } n/2 + 1 \le i \le n \end{cases}$

Define f*: $E(G) \rightarrow \{0,1\}$

For j = 1,3…m

f*(eij) = $\begin{cases} 1 & \text{for } 1 \le i \le n/2 \\ 0 & \text{for } (n/2) + 1 \le i \le n \end{cases}$

For j = 2,4…m-1

f*(eij) = $\begin{cases} 0 & \text{for } 1 \le i \le n/2 \\ 1 & \text{for } (n/2) + 1 \le i \le n \end{cases}$

Then for each vertex vi,

Let T = f(vi) +$\sum_{j=1}^{m}$ f*(eij)

$$= \begin{cases} 0 + \frac{m+1}{2} & \text{for } 1 \le i \le \frac{n}{2} \\ 1 + 0 + \frac{m-1}{2} & \text{for } \left(\frac{n}{2}\right) + 1 \le i \le n \end{cases}$$

= (m+1)/2 for all 1≤ i ≤ n

Here g(0) = v(0) + e(0) = n/2 + (mn/2) = (m + 1)n/2: g(1) = v(1) + e(1) = n/2 + (mn/2) = (m + 1)n/2

│g(0)-g(1)│= 0 ≤ 1. Therefore the condition is hold.

Case (ii): When n is odd

Define f: V(G)→{0,1}as

f(vi) = $\begin{cases} 0 & \text{for } 1 \le i \le \frac{n-1}{2} \\ 1 & \text{for} \frac{n+1}{2} + \le i \le n \end{cases}$

Define f*: $E(G) \rightarrow \{0,1\}$

For j = 1,3…m

f*(eij) = $\begin{cases} 1 & \text{for } 1 \le i \le \frac{n-1}{2} \\ 0 & \text{for} \frac{n+1}{2} \le i \le n \end{cases}$

For j = 2,4…m-1

f*(eij) = $\begin{cases} 0 & \text{for } 1 \le i \le \frac{n-1}{2} \\ 1 & \text{for} \frac{n+1}{2} \le i \le n \end{cases}$

For every vertex vi

T = f(vi) +$\sum_{j=1}^{m}$ f*(eij)

$$= \begin{cases} 0 + \frac{m+1}{2} & \text{for } 1 \le i \le \frac{n-1}{2} \\ 1 + 0 + \frac{m-1}{2} & \text{for } \frac{n+1}{2} \le i \le n \end{cases}$$

=(m+1)/2 for all 1≤ i ≤ n

Here g(0) = v(0) + e(0); Where v(0)= (n-1)/2; e(0) = [(m-1)/2][(n-1)/2]+ [(m+1)/2][(n+1)/2] = (mn + 1)/2;

g(0) =(1/2)[n-1+mn+1] =(1/2)(m+1)n;

g(1) = v(1) + e(1); Where v(1)= (n+1)/2; e(1) = [(m+1)/2][(n-1)/2]+ [(m-1)/2][(n+1)/2] = (mn - 1)/2;

g(1) =(1/2)[n+1+mn-1] =(1/2)(m+1)n

│g(0)-g(1)│= 0 ≤ 1.Therefore the condition is satisfied.

Hence the odd regular direted graph is total magic cordial.

IV. CONCLUSION AND OPEN PROBLEMS

We proved that the regular directed graph is edge product cordial and the odd regular directed graph is total magic cordial. In future one can investigate whether these two labelings exist on irregular and oriented digraphs or not and if exists, we can obtain the necessary condition at which they are cordial.

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